

# Contents

<b>1</b>	<b>Resolutions of the ground ring</b>	<b>3</b>
<b>2</b>	<b>Resolutions of modules</b>	<b>4</b>
<b>3</b>	<b>Induced equivariant chain maps</b>	<b>5</b>
<b>4</b>	<b>Functors</b>	<b>6</b>
<b>5</b>	<b>Chain complexes</b>	<b>7</b>
<b>6</b>	<b>Homology and cohomology groups</b>	<b>8</b>
<b>7</b>	<b>Poincare series</b>	<b>9</b>
<b>8</b>	<b>Cohomology ring structure</b>	<b>10</b>
<b>9</b>	<b>Commutator and nonabelian tensor computations</b>	<b>11</b>
<b>10</b>	<b>Lie commutators and nonabelian Lie tensors</b>	<b>12</b>
<b>11</b>	<b>Generators and relators of groups</b>	<b>13</b>
<b>12</b>	<b>Orbit polytopes and fundamental domains</b>	<b>14</b>
<b>13</b>	<b>Cocycles</b>	<b>15</b>
<b>14</b>	<b>Words in free <math>ZG</math>-modules</b>	<b>16</b>
<b>15</b>	<b><math>FpG</math>-modules</b>	<b>17</b>
<b>16</b>	<b>Meataxe modules</b>	<b>18</b>
<b>17</b>	<b>G-Outer Groups</b>	<b>19</b>
<b>18</b>	<b>Cat-1-groups</b>	<b>20</b>
<b>19</b>	<b>Coxeter diagrams and graphs of groups</b>	<b>21</b>
<b>20</b>	<b>Some functions for accessing basic data</b>	<b>22</b>

<b>21 Parallel Computation - Core Functions</b>	<b>23</b>
<b>22 Parallel Computation - Extra Functions</b>	<b>24</b>
<b>23 Topological Data Analysis</b>	<b>25</b>
<b>24 Pseudo lists</b>	<b>26</b>
<b>25 Miscellaneous</b>	<b>27</b>

# Chapter 1

## Resolutions of the ground ring

`ResolutionAbelianGroup(L,n)`   `ResolutionAbelianGroup(G,n)` Inputs a list  $L := [m_1, m_2, \dots, m_d]$  of nonneg  
`ResolutionAlmostCrystalGroup(G,n)` Inputs a positive integer  $n$  and an almost crystallographic pcg group  $G$ . I  
`ResolutionAlmostCrystalQuotient(G,n,c)`   `ResolutionAlmostCrystalQuotient(G,n,c,false)` An alm  
`ResolutionArtinGroup(D,n)` Inputs a Coxeter diagram  $D$  and an integer  $n > 1$ . It returns  $n$  terms of a free  $ZG$ -re  
`ResolutionAsphericalPresentation(F,R,n)` Inputs a free group  $F$ , a set  $R$  of words in  $F$  which constitute an a  
`ResolutionBieberbachGroup( G )`  
`ResolutionBieberbachGroup( G, v )` Inputs a Bieberbach group  $G$  (represented using `AffineCrystGroupOnR`  
`ResolutionDirectProduct(R,S)`   `ResolutionDirectProduct(R,S,"internal")` Inputs a  $ZG$ -resolution  $R$   
`ResolutionExtension(g,R,S)`   `ResolutionExtension(g,R, S, "TestFiniteness")`   `ResolutionExter`  
`ResolutionFiniteDirectProduct(R,S)`   `ResolutionFiniteDirectProduct(R,S, "internal")` Inputs a  
`ResolutionFiniteExtension(gensE,gensG,R,n)`   `ResolutionFiniteExtension(gensE,gensG,R,n,true)`  
`ResolutionFiniteGroup(gens,n)`   `ResolutionFiniteGroup(gens,n,true)`   `ResolutionFiniteGroup(g`  
`ResolutionFiniteSubgroup(R,K)`   `ResolutionFiniteSubgroup(R,gensG,gensK)` Inputs a  $ZG$ -resolution fo  
`ResolutionGraphOfGroups(D,n)`   `ResolutionGraphOfGroups(D,n,L)` Inputs a graph of groups  $D$  and a p  
`ResolutionNilpotentGroup(G,n)`   `ResolutionNilpotentGroup(G,n, "TestFiniteness")` Inputs a nilpot  
`ResolutionNormalSeries(L,n)`   `ResolutionNormalSeries(L,n,true)`   `ResolutionNormalSeries(L,n`  
`ResolutionPrimePowerGroup(P,n)`   `ResolutionPrimePowerGroup(G,n,p)` Inputs a  $p$ -group  $P$  and integer  
`ResolutionSmallFpGroup(G,n)`   `ResolutionSmallFpGroup(G,n,p)` Inputs a small finitely presented group  
`ResolutionSubgroup(R,K)` Inputs a  $ZG$ -resolution for an (infinite) group  $G$  and a subgroup  $K$  of finite index  $|G :$   
`ResolutionSubnormalSeries(L,n)` Inputs a positive integer  $n$  and a list  $L = [L_1, \dots, L_k]$  of subgroups  $L_i$  of a fin  
`TwistedTensorProduct(R,S,EhomG,GmapE,NhomE,NEhomN,Eltse,Mult,InvE)` Inputs a  $ZG$ -resolution  $R$ , a  $ZN$

## Chapter 2

# Resolutions of modules

| `ResolutionFpGModule (M, n)` Inputs an  $FpG$ -module  $M$  and a positive integer  $n$ . It returns  $n$  terms of a minimal fr

## Chapter 3

# Induced equivariant chain maps

| `EquivariantChainMap(R, S, f)` Inputs a  $ZG$ -resolution  $R$ , a  $ZG'$ -resolution  $S$ , and a group homomorphism  $f : G \rightarrow G'$

## Chapter 4

# Functors

HomToIntegers( $X$ ) Inputs either a  $ZG$ -resolution  $X = R$ , or an equivariant chain map  $X = (F : R \longrightarrow S)$ . It returns the chain complex obtained by applying  $\text{Hom}_{\mathbb{Z}}(-, \mathbb{Z})$  to the resolution  $R$ .

HomToIntegersModP( $R$ ) Inputs a  $ZG$ -resolution  $R$  and returns the cochain complex obtained by applying  $\text{Hom}_{\mathbb{Z}}(-, \mathbb{Z})$  to the resolution  $R$ .

HomToIntegralModule( $R, f$ ) Inputs a  $ZG$ -resolution  $R$  and a group homomorphism  $f : G \longrightarrow GL_n(\mathbb{Z})$  to the group  $G$ . It returns the chain complex obtained by applying  $\text{Hom}_{\mathbb{Z}}(-, \mathbb{Z})$  to the resolution  $R$ .

HomToGModule( $R, A$ ) Inputs a  $ZG$ -resolution  $R$  and an abelian  $G$ -outer group  $A$ . It returns the  $G$ -cocomplex obtained by applying  $\text{Hom}_{\mathbb{Z}}(-, A)$  to the resolution  $R$ .

LowerCentralSeriesLieAlgebra( $G$ ) LowerCentralSeriesLieAlgebra( $f$ ) Inputs a pcg group  $G$ . If each  $g \in G$  is represented by a matrix  $f(g) \in GL_n(\mathbb{Z})$ , then the Lie algebra of  $G$  is the Lie algebra of the matrices  $f(g)$ .

TensorWithIntegers( $X$ ) Inputs either a  $ZG$ -resolution  $X = R$ , or an equivariant chain map  $X = (F : R \longrightarrow S)$ . It returns the chain complex obtained by tensoring  $R$  with  $\mathbb{Z}$ .

TensorWithIntegersModP( $X, p$ ) Inputs either a  $ZG$ -resolution  $X = R$ , or an equivariant chain map  $X = (F : R \longrightarrow S)$ . It returns the chain complex obtained by tensoring  $R$  with  $\mathbb{Z}/p\mathbb{Z}$ .

TensorWithRationals( $R$ ) Inputs a  $ZG$ -resolution  $R$  and returns the chain complex obtained by tensoring  $R$  with  $\mathbb{Q}$ .

## Chapter 5

# Chain complexes

`ChevalleyEilenbergComplex(X, n)` Inputs either a Lie algebra  $X = A$  (over the ring of integers  $Z$  or over a field  $K$ ) or a Leibniz algebra  $X = A$  (over the ring of integers  $Z$  or over a field  $K$ )

`LeibnizComplex(X, n)` Inputs either a Lie or Leibniz algebra  $X = A$  (over the ring of integers  $Z$  or over a field  $K$ )

## Chapter 6

# Homology and cohomology groups

`Cohomology(X, n)` Inputs either a cochain complex  $X = C$  (or  $G$ -cocomplex  $C$ ) or a cochain map  $X = (C \longrightarrow D)$  or  
`CohomologyModule(C, n)` Inputs a  $G$ -cocomplex  $C$  together with a non-negative integer  $n$ . It returns the cohomology  
`CohomologyPrimePart(C, n, p)` Inputs a cochain complex  $C$  in characteristic 0, a positive integer  $n$ , and a prime  $p$ .  
`GroupCohomology(X, n)` `GroupCohomology(X, n, p)` Inputs a positive integer  $n$  and either a finite group  $X = G$  or  
`GroupHomology(X, n)`  
`GroupHomology(X, n, p)` Inputs a positive integer  $n$  and either a finite group  $X = G$  or a Coxeter diagram  $X = D$  representing  
`Homology(X, n)` Inputs either a chain complex  $X = C$  or a chain map  $X = (C \longrightarrow D)$ . If  $X = C$  then the torsion coefficients  
`HomologyPb(C, n)` This is a back-up function which might work in some instances where `Homology(C, n)` fails. It is  
`HomologyPrimePart(C, n, p)` Inputs a chain complex  $C$  in characteristic 0, a positive integer  $n$ , and a prime  $p$ . It returns  
`LeibnizAlgebraHomology(A, n)` Inputs a Lie or Leibniz algebra  $X = A$  (over the ring of integers  $Z$  or over a field  $K$ ) and a positive integer  $n$ .  
`LieAlgebraHomology(A, n)` Inputs a Lie algebra  $A$  (over the integers or a field) and a positive integer  $n$ . It returns the  
`PrimePartDerivedFunctor(G, R, F, n)` Inputs a finite group  $G$ , a positive integer  $n$ , at least  $n + 1$  terms of a  $ZP$ -resolution  
`RankHomologyPGroup(G, n)` `RankHomologyPGroup(R, n)` `RankHomologyPGroup(G, n, "empirical")` Inputs a finite group  $G$ , a positive integer  $n$ , and  
`RankPrimeHomology(G, n)` Inputs a (smallish)  $p$ -group  $G$  together with a positive integer  $n$ . It returns a function `dim`



## Chapter 7

### Poincare series

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`EfficientNormalSubgroups(G)`

`EfficientNormalSubgroups(G, k)` Inputs a prime-power group  $G$  and, optionally, a positive integer  $k$ . The default

`ExpansionOfRationalFunction(f, n)` Inputs a positive integer  $n$  and a rational function  $f(x) = p(x)/q(x)$  where  $t$

`PoincareSeries(G, n)` `PoincareSeries(R, n)`

`PoincareSeries(L, n)`

`PoincareSeries(G)` Inputs a finite  $p$ -group  $G$  and a positive integer  $n$ . It returns a quotient of polynomials  $f(x) =$

`PoincareSeriesPrimePart(G, p, n)` Inputs a finite group  $G$ , a prime  $p$ , and a positive integer  $n$ . It returns a quotient

`Prank(G)` Inputs a  $p$ -group  $G$  and returns the rank of the largest elementary abelian subgroup.

## Chapter 8

# Cohomology ring structure

`IntegralCupProduct(R, u, v, p, q)`  
`IntegralCupProduct(R, u, v, p, q, P, Q, N)` (Various functions used to construct the cup product are also available)  
`IntegralRingGenerators(R, n)` Inputs at least  $n + 1$  terms of a  $ZG$ -resolution and integer  $n > 0$ . It returns a minimal  
`ModPCohomologyGenerators(G, n)`  
`ModPCohomologyGenerators(R)` Inputs either a  $p$ -group  $G$  and positive integer  $n$ , or else  $n$  terms of a minimal  $ZG$ -resolution  
`ModPCohomologyRing(G, n)`  
`ModPCohomologyRing(G, n, level)`  
`ModPCohomologyRing(R)`  
`ModPCohomologyRing(R, level)` Inputs either a  $p$ -group  $G$  and positive integer  $n$ , or else  $n$  terms of a minimal  $ZG$ -resolution  
`ModPRingGenerators(A)` Inputs a mod  $p$  cohomology ring  $A$  (created using the preceding function). It returns a minimal

## Chapter 9

# Commutator and nonabelian tensor computations

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`BaerInvariant(G, c)` Inputs a nilpotent group  $G$  and integer  $c > 0$ . It returns the Baer invariant  $M^{(c)}(G)$  defined as  $M^{(c)}(G) = \gamma_c(G) / \gamma_{c+1}(G)$ .

`Coclass(G)` Inputs a group  $G$  of prime-power order  $p^n$  and nilpotency class  $c$  say. It returns the integer  $r = n - c$ .

`EpiCentre(G, N)` Inputs a finite group  $G$  and normal subgroup  $N$  and returns a subgroup  $Z^*(G, N)$  of the centre of  $N$ .

`NonabelianExteriorProduct(G, N)` Inputs a finite group  $G$  and normal subgroup  $N$ . It returns a record  $E$  with the following fields:

- `NonabelianSymmetricKernel(G)`
- `NonabelianSymmetricKernel(G, m)` Inputs a finite or nilpotent infinite group  $G$  and returns the abelian invariant  $\gamma_m(G)$ .
- `NonabelianSymmetricSquare(G)`
- `NonabelianSymmetricSquare(G, m)` Inputs a finite or nilpotent infinite group  $G$  and returns a record  $T$  with the following fields:

`NonabelianTensorProduct(G, N)` Inputs a finite group  $G$  and normal subgroup  $N$ . It returns a record  $E$  with the following fields:

- `NonabelianTensorSquare(G)`
- `NonabelianTensorSquare(G, m)` Inputs a finite or nilpotent infinite group  $G$  and returns a record  $T$  with the following fields:

`RelativeSchurMultiplier(G, N)` Inputs a finite group  $G$  and normal subgroup  $N$ . It returns the homology group  $H_2(G/N, \mathbb{Z})$ .

`TensorCentre(G)` Inputs a group  $G$  and returns the largest central subgroup  $N$  such that the induced homomorphism  $G/N \rightarrow G/N$  is trivial.

`ThirdHomotopyGroupOfSuspensionB(G)`

`ThirdHomotopyGroupOfSuspensionB(G, m)` Inputs a finite or nilpotent infinite group  $G$  and returns the abelian invariant  $\gamma_m(G)$ .

`UpperEpicentralSeries(G, c)` Inputs a nilpotent group  $G$  and an integer  $c$ . It returns the  $c$ -th term of the upper epicentral series of  $G$ .

## Chapter 10

# Lie commutators and nonabelian Lie tensors

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Functions on this page are joint work with Hamid Mohammadzadeh, and implemented by him.

`LieCoveringHomomorphism(L)` Inputs a finite dimensional Lie algebra  $L$  over a field, and returns a surjective Lie

`LeibnizQuasiCoveringHomomorphism(L)` Inputs a finite dimensional Lie algebra  $L$  over a field, and returns a sur

`LieEpiCentre(L)` Inputs a finite dimensional Lie algebra  $L$  over a field, and returns an ideal  $Z^*(L)$  of the centre of

`LieExteriorSquare(L)` Inputs a finite dimensional Lie algebra  $L$  over a field. It returns a record  $E$  with the follow

`LieTensorSquare(L)` Inputs a finite dimensional Lie algebra  $L$  over a field and returns a record  $T$  with the follow

`LieTensorCentre(L)` Inputs a finite dimensional Lie algebra  $L$  over a field and returns the largest ideal  $N$  such th

## Chapter 11

# Generators and relators of groups

•  
`CayleyGraphDisplay(G,X)`

`CayleyGraphDisplay(G,X,"mozilla")` Inputs a finite group  $G$  together with a subset  $X$  of  $G$ . It displays the co

`IdentityAmongRelatorsDisplay(R,n)` `IdentityAmongRelatorsDisplay(R,n,"mozilla")` Inputs a free

`IsAspherical(F,R)` Inputs a free group  $F$  and a set  $R$  of words in  $F$ . It performs a test on the 2-dimensional CW

`PresentationOfResolution(R)` Inputs at least two terms of a reduced  $ZG$ -resolution  $R$  and returns a record  $P$  w

`TorsionGeneratorsAbelianGroup(G)` Inputs an abelian group  $G$  and returns a generating set  $[x_1, \dots, x_n]$  where

## Chapter 12

# Orbit polytopes and fundamental domains

•  
FundamentalDomainAffineCrystGroupOnRight( $v, G$ ) Inputs a crystallographic group  $G$  (represented using Affi  
OrbitPolytope( $G, v, L$ ) Inputs a permutation group or matrix group  $G$  of degree  $n$  and a rational vector  $v$  of leng  
PolytopalComplex( $G, v$ )  
PolytopalComplex( $G, v, n$ ) Inputs a permutation group or matrix group  $G$  of degree  $n$  and a rational vector  $v$  of  
PolytopalGenerators( $G, v$ ) Inputs a permutation group or matrix group  $G$  of degree  $n$  and a rational vector  $v$  of  
VectorStabilizer( $G, v$ ) Inputs a permutation group or matrix group  $G$  of degree  $n$  and a rational vector of degr

## Chapter 13

### Cocycles

•

`CcGroup(A, f)` Inputs a  $G$ -module  $A$  (i.e. an abelian  $G$ -outer group) and a standard 2-cocycle  $f: G \times G \rightarrow A$ . It returns an integer matrix  $M$  with the following property:  $M \cdot M^T = 0$ .

`CocycleCondition(R, n)` Inputs a resolution  $R$  and an integer  $n > 0$ . It returns an integer matrix  $M$  with the following property:  $M \cdot M^T = 0$ .

`StandardCocycle(R, f, n)` Inputs a  $ZG$ -resolution  $R$  (with contracting homotopy), a positive integer  $n$  and an integer  $f$ . It returns an integer matrix  $M$  with the following property:  $M \cdot M^T = 0$ .

`Syzygy(R, g)` Inputs a  $ZG$ -resolution  $R$  (with contracting homotopy) and a list  $g = [g[1], \dots, g[n]]$  of elements in  $G$ . It returns an integer matrix  $M$  with the following property:  $M \cdot M^T = 0$ .

## Chapter 14

### Words in free $ZG$ -modules

`AddFreeWords(v, w)` Inputs two words  $v, w$  in a free  $ZG$ -module and returns their sum  $v + w$ . If the characteristic of  $Z$  is  $p$ , then  $v + w$  is computed modulo  $p$ .

`AddFreeWordsModP(v, w, p)` Inputs two words  $v, w$  in a free  $ZG$ -module and the characteristic  $p$  of  $Z$ . It returns the sum  $v + w$  modulo  $p$ .

`AlgebraicReduction(w)` Inputs a word  $w$  in a free  $ZG$ -module and returns a reduced version of the word in  $w$ .

`AlgebraicReduction(w, p)` Inputs a word  $w$  in a free  $ZG$ -module and returns a reduced version of the word in  $w$  modulo  $p$ .

`Multiply Word(n, w)` Inputs a word  $w$  and integer  $n$ . It returns the scalar multiple  $n \cdot w$ .

`Negate([i, j])` Inputs a pair  $[i, j]$  of integers and returns  $[-i, j]$ .

`NegateWord(w)` Inputs a word  $w$  in a free  $ZG$ -module and returns the negated word  $-w$ .

`PrintZGword(w, elts)` Inputs a word  $w$  in a free  $ZG$ -module and a (possibly partial but sufficient) listing `elts` of the elements of  $G$ . It prints the word  $w$  in terms of the elements of  $G$ .

`TietzeReduction(S, w)` Inputs a set  $S$  of words in a free  $ZG$ -module, and a word  $w$  in the module. The function returns a reduced version of  $w$  modulo  $S$ .



## Chapter 15

### *FpG*-modules

`DirectSumOfFpGModules (M, N)`  
`DirectSumOfFpGModules ([ M[1], M[2], ..., M[k] ])` Inputs two *FpG*-modules  $M$  and  $N$  with common group  $G$ .  
`FpGModule (A, P)`  
`FpGModule (A, G, p)` Inputs a  $p$ -group  $P$  and a matrix  $A$  whose rows have length a multiple of the order of  $G$ . It returns an *FpG*-module.  
`FpGModuleDualBasis (M)` Inputs an *FpG*-module  $M$ . It returns a record  $R$  with two components:  $R.freeModule$  is a free module and  $R.dualBasis$  is a dual basis.  
`FpGModuleHomomorphism (M, N, A)`  
`FpGModuleHomomorphismNC (M, N, A)` Inputs *FpG*-modules  $M$  and  $N$  over a common  $p$ -group  $G$ . Also inputs a list  $A$  of matrices.  
`DesuspensionFpGModule (M, n)`  
`DesuspensionFpGModule (R, n)` Inputs a positive integer  $n$  and an *FpG*-module  $M$ . It returns an *FpG*-module  $D^n M$ .  
`RadicalOfFpGModule (M)` Inputs an *FpG*-module  $M$  with  $G$  a  $p$ -group, and returns the Radical of  $M$  as an *FpG*-module.  
`GeneratorsOfFpGModule (M)` Inputs an *FpG*-module  $M$  and returns a matrix whose rows correspond to a minimal generating set.  
`ImageOfFpGModuleHomomorphism (f)` Inputs an *FpG*-module homomorphism  $f : M \rightarrow N$  and returns its image.  
`IntersectionOfFpGModules (M, N)` Inputs two *FpG*-modules  $M, N$  arising as submodules in a common free module.  
`IsFpGModuleHomomorphismData (M, N, A)` Inputs *FpG*-modules  $M$  and  $N$  over a common  $p$ -group  $G$ . Also inputs a list  $A$  of matrices.  
`MultipleOfFpGModule (w, M)` Inputs an *FpG*-module  $M$  and a list  $w := [g_1, \dots, g_t]$  of elements in the group  $G = M$ .  
`ProjectedFpGModule (M, k)` Inputs an *FpG*-module  $M$  of ambient dimension  $n|G|$ , and an integer  $k$  between 1 and  $n$ .  
`RandomHomomorphismOfFpGModules (M, N)` Inputs two *FpG*-modules  $M$  and  $N$  over a common group  $G$ . It returns a random homomorphism.  
`Rank (f)` Inputs an *FpG*-module homomorphism  $f : M \rightarrow N$  and returns the dimension of the image of  $f$  as a vector space.  
`SumOfFpGModules (M, N)` Inputs two *FpG*-modules  $M, N$  arising as submodules in a common free module  $(FG)^n$ .  
`SumOp (f, g)` Inputs two *FpG*-module homomorphisms  $f, g : M \rightarrow N$  with common source and common target. It returns  $f + g$ .  
`VectorsToFpGModuleWords (M, L)` Inputs an *FpG*-module  $M$  and a list  $L = [v_1, \dots, v_k]$  of vectors in  $M$ . It returns a list of words.

## Chapter 16

### Meataxe modules

•  
DesuspensionMtxModule( $M$ ) Inputs a meataxe module  $M$  over the field of  $p$  elements and returns an FpG-module  $I$ .  
FpG\_to\_MtxModule( $M$ ) Inputs an FpG-module  $M$  and returns an isomorphic meataxe module.  
GeneratorsOfMtxModule( $M$ ) Inputs a meataxe module  $M$  acting on, say, the vector space  $V$ . The function returns

## Chapter 17

# G-Outer Groups

`GOuterGroup(E, N)`

`GOuterGroup()` Inputs a group  $E$  and normal subgroup  $N$ . It returns  $N$  as a  $G$ -outer group where  $G = E/N$ . The fun

`GOuterGroupHomomorphismNC(A, B, phi)`

`GOuterGroupHomomorphismNC()` Inputs  $G$ -outer groups  $A$  and  $B$  with common acting group, and a group homomor

`GOuterHomomorphismTester(A, B, phi)` Inputs  $G$ -outer groups  $A$  and  $B$  with common acting group, and a group ho

`Centre(A)` Inputs  $G$ -outer group  $A$  and returns the group theoretic centre of `ActedGroup(A)` as a  $G$ -outer group.

`DirectProductGog(A, B)`

`DirectProductGog(Lst)` Inputs  $G$ -outer groups  $A$  and  $B$  with common acting group, and returns their group-theore

## Chapter 18

### Cat-1-groups

`AutomorphismGroupAsCatOneGroup(G)` Inputs a group  $G$  and returns the Cat-1-group  $C$  corresponding to the crossed module  $(G, \text{triv})$ .

`HomotopyGroup(C, n)` Inputs a cat-1-group  $C$  and an integer  $n$ . It returns the  $n$ th homotopy group of  $C$ .

`HomotopyModule(C, 2)` Inputs a cat-1-group  $C$  and an integer  $n=2$ . It returns the second homotopy group of  $C$  as a  $C$ -module.

`ModuleAsCatOneGroup(G, alpha, M)` Inputs a group  $G$ , an abelian group  $M$  and a homomorphism  $\alpha: G \rightarrow \text{Aut}(M)$ . It returns the Cat-1-group  $C$  corresponding to the crossed module  $(M, \alpha)$ .

`MooreComplex(C)` Inputs a cat-1-group  $C$  and returns its Moore complex  $[M_1 \rightarrow M_0]$  as a list whose single entry is the Moore complex.

`NormalSubgroupAsCatOneGroup(G, N)` Inputs a group  $G$  with normal subgroup  $N$ . It returns the Cat-1-group  $C$  corresponding to the crossed module  $(N, \text{triv})$ .

## Chapter 19

# Coxeter diagrams and graphs of groups

`CoxeterDiagramComponents(D)` Inputs a Coxeter diagram  $D$  and returns a list  $[D_1, \dots, D_d]$  of the maximal connected components of  $D$ .

`CoxeterDiagramDegree(D, v)` Inputs a Coxeter diagram  $D$  and vertex  $v$ . It returns the degree of  $v$  (i.e. the number of edges incident to  $v$ ).

`CoxeterDiagramDisplay(D)` Inputs a Coxeter diagram  $D$  and displays it as a .gif file. It uses the web browser.

`CoxeterDiagramDisplay(D, "web browser")` Inputs a Coxeter diagram  $D$  and displays it as a .gif file. It uses the web browser.

`CoxeterDiagramFpArtinGroup(D)` Inputs a Coxeter diagram  $D$  and returns the corresponding finitely presented Artin group.

`CoxeterDiagramFpCoxeterGroup(D)` Inputs a Coxeter diagram  $D$  and returns the corresponding finitely presented Coxeter group.

`CoxeterDiagramIsSpherical(D)` Inputs a Coxeter diagram  $D$  and returns "true" if the associated Coxeter group is spherical.

`CoxeterDiagramMatrix(D)` Inputs a Coxeter diagram  $D$  and returns a matrix representation of it. The matrix is the Coxeter matrix.

`CoxeterSubDiagram(D, V)` Inputs a Coxeter diagram  $D$  and a subset  $V$  of its vertices. It returns the full sub-diagram induced by  $V$ .

`CoxeterDiagramVertices(D)` Inputs a Coxeter diagram  $D$  and returns its set of vertices.

`EvenSubgroup(G)` Inputs a group  $G$  and returns a subgroup  $G^+$ . The subgroup is that generated by all products  $xy$  where  $x, y$  are elements of  $G$ .

`GraphOfGroupsDisplay(D)` Inputs a graph of groups  $D$  and displays it as a .gif file. It uses the web browser.

`GraphOfGroupsDisplay(D, "web browser")` Inputs a graph of groups  $D$  and displays it as a .gif file. It uses the web browser.

`GraphOfGroupsTest(D)` Inputs an object  $D$  and tries to test whether it is a Graph of Groups. However, it DOES NOT work.

## Chapter 20

### Some functions for accessing basic data

`BoundaryMap(C)` Inputs a resolution, chain complex or cochain complex  $C$  and returns the function  $C!.boundary$ .  
`BoundaryMatrix(C,n)` Inputs a chain or cochain complex  $C$  and integer  $n>0$ . It returns the  $n$ -th boundary map of  $C$ .  
`Dimension(C)` Inputs a resolution, chain complex or cochain complex  $C$  and returns the function  $C!.dimension$ .  
`Dimension(M)` Inputs a resolution, chain complex or cochain complex  $C$  and returns the function  $C!.dimension$ .  
`EvaluateProperty(X,"name")` Inputs a component object  $X$  (such as a  $ZG$ -resolution or chain map) and a string  $name$ .  
`GroupOfResolution(R)` Inputs a  $ZG$ -resolution  $R$  and returns the group  $G$ .  
`Length(R)` Inputs a resolution  $R$  and returns its length (i.e. the number of terms of  $R$  that HAP has computed).  
`Map(f)` Inputs a chain map, or cochain map or equivariant chain map  $f$  and returns the mapping function (as opposed to the mapping object).  
`Source(f)` Inputs a chain map, or cochain map, or equivariant chain map, or  $FpG$ -module homomorphism  $f$  and returns the source module.  
`Target(f)` Inputs a chain map, or cochain map, or equivariant chain map, or  $FpG$ -module homomorphism  $f$  and returns the target module.

## Chapter 21

# Parallel Computation - Core Functions

`ChildProcess()`

`ChildProcess("computer.ac.wales")` This starts a GAP session as a child process and returns a stream to the child process.

- open a shell on thishost
- cd .ssh
- ls
- > if id\_dsa, id\_rsa etc exists, skip the next two steps!
- ssh-keygen -t rsa
- ssh-keygen -t dsa
- scp \*.pub user@remotehost:~/
- ssh remotehost -l user
- cat id\_rsa.pub >> .ssh/authorized\_keys
- cat id\_dsa.pub >> .ssh/authorized\_keys
- rm id\_rsa.pub id\_dsa.pub
- exit

You should now be able to connect from "thishost" to "remotehost" without a password prompt.)

`ChildClose(s)` This closes the stream `s` to a child GAP process.

`ChildCommand("cmd;", s)` This runs a GAP command "cmd;" on the child process accessed by the stream `s`. Here `s` is a stream to a child process.

`NextAvailableChild(L)` Inputs a list `L` of child processes and returns a child in `L` which is ready for computation.

`IsAvailableChild(s)` Inputs a child process `s` and returns true if `s` is currently available for computations, and false otherwise.

`ChildPut(A, "B", s)` This copies a GAP object `A` on the parent process to an object `B` on the child process `s`. (The object `B` is created on the child process.)

`ChildGet("A", s)` This function copies a GAP object `A` on the child process `s` and returns it on the parent process.

`HAPPrintTo("file", R)` Inputs a name "file" of a new text file and a HAP object `R`. It writes the object `R` to "file".

`HAPRead("file", R)` Inputs a name "file" containing a HAP object `R` and returns the object. Currently this is only implemented for text files.

## Chapter 22

# Parallel Computation - Extra Functions

`ChildFunction("function(arg);", s)` This runs the GAP function "function(arg);" on a child process accessed by `s`.  
`ChildRead(s)` This returns, as a string, the output of the last application of `ChildFunction("function(arg);", s)`.  
`ChildReadEval(s)` This returns, as an evaluated string, the output of the last application of `ChildFunction("function(arg);", s)`.  
`ParallelList(I, fn, L)` Inputs a list  $I$ , a function  $fn$  such that  $fn(x)$  is defined for all  $x$  in  $I$ , and a list of children  $L$ .



## Chapter 23

# Topological Data Analysis

`MatrixToTopologicalSpace(A,n)` Inputs an integer matrix  $A$  and an integer  $n$ . It returns a 2-dimensional topological space.

`ReadImageAsTopologicalSpace("file.png",n)` `ReadImageAsTopologicalSpace("file.png",[m,n])` Reads an image file ("file.png", "file.eps", "file.bmp" etc) and returns an integer matrix of size  $[m,n]$ .

`ReadImageAsMatrix("file.png")` Reads an image file ("file.png", "file.eps", "file.bmp" etc) and returns an integer matrix of size  $[m,n]$ .

`WriteTopologicalSpaceAsImage(T,"filename","ext")` Inputs a 2-dimensional topological space  $T$ , and a file name "filename" and extension "ext". It writes the image of  $T$  to the file "filename.ext".

`ViewTopologicalSpace(T)` `ViewTopologicalSpace(T,"mozilla")` Inputs a topological space  $T$ , and optional browser "mozilla". It displays the image of  $T$  in the browser.

`Bettinnumbers(T,n)` `Bettinnumbers(T)` Inputs a topological space  $T$  and a non-negative integer  $n$ . It returns the  $n$ -th Bettin number of  $T$ .

`PathComponent(T,n)` Inputs a topological space  $T$  and an integer  $n$  in the range  $0, \dots, \text{Bettinnumbers}(T,0)$ . It returns the  $n$ -th path component of  $T$ .

`SingularChainComplex(A)` Inputs a topological space  $T$  and returns a (usually very large) integral chain complex of  $T$ .

`ContractTopologicalSpace(T)` Inputs a topological space  $T$  of dimension  $d$  and removes  $d$ -dimensional cells from  $T$ .

`BoundaryTopologicalSpace(T)` Inputs a topological space  $T$  and returns its boundary as a topological space.

`BoundarySingularities(T)` Inputs a topological space  $T$  and returns the subspace of points in the boundary where the boundary is singular.

`ThickenedTopologicalSpace(T)` `ThickenedTopologicalSpace(T,n)` Inputs a topological space  $T$  and returns the  $n$ -th thickened space of  $T$ .

`ComplementTopologicalSpace(T)` Inputs a topological space  $T$  and returns a topological space  $S$ . A euclidean point  $x$  is in  $S$  if and only if  $x$  is not in  $T$ .

`ConcatenatedTopologicalSpace(L)` Inputs a list  $L$  of topological spaces whose underlying arrays of numbers are all the same size. It returns the concatenation of the spaces in  $L$ .

## Chapter 24

### Pseudo lists

`Add(L, x)` Let  $L$  be a pseudo list of length  $n$ , and  $x$  an object compatible with the entries in  $L$ . If  $x$  is not in  $L$  then th  
`Append(L, K)` Let  $L$  be a pseudo list and  $K$  a list whose objects are compatible with those in  $L$ . This operation appli  
`ListToPseudoList(L)` Inputs a list  $L$  and returns the pseudo list representation of  $L$ .

## Chapter 25

### Miscellaneous

`BigStepLCS(G, n)` Inputs a group  $G$  and a positive integer  $n$ . It returns a subseries  $G = L_1 > L_2 > \dots L_k = 1$  of the 1  
`Compose(f, g)` Inputs two  $FpG$ -module homomorphisms  $f : M \longrightarrow N$  and  $g : L \longrightarrow M$  with  $Source(f) = Target(g)$   
`HAPcopyright()` This function provides details of HAP'S GNU public copyright licence.  
`IsLieAlgebraHomomorphism(f)` Inputs an object  $f$  and returns true if  $f$  is a homomorphism  $f : A \longrightarrow B$  of Lie algebras  
`IsSuperperfect(G)` Inputs a group  $G$  and returns "true" if both the first and second integral homology of  $G$  is trivial  
`MakeHAPManual()` This function creates the manual for HAP from an XML file.  
`PermToMatrixGroup(G, n)` Inputs a permutation group  $G$  and its degree  $n$ . Returns a bijective homomorphism  $f : G \longrightarrow GL(n, \mathbb{C})$   
`SolutionsMatDestructive(M, B)` Inputs an  $m \times n$  matrix  $M$  and a  $k \times n$  matrix  $B$  over a field. It returns a  $k \times m$  matrix  
`TestHap()` This runs a representative sample of HAP functions and checks to see that they produce the correct output

# Index

- Add, [26](#)
- AddFreeWords, [16](#)
- AddFreeWordsModP, [16](#)
- AlgebraicReduction, [16](#)
- Append, [26](#)
- AutomorphismGroupAsCatOneGroup, [20](#)
  
- BaerInvariant, [11](#)
- Bettinnumbers, [25](#)
- BigStepLCS, [27](#)
- BoundaryMap, [22](#)
- BoundaryMatrix, [22](#)
- BoundarySingularities, [25](#)
- BoundaryTopologicalSpace, [25](#)
  
- CayleyGraphDisplay, [13](#)
- CcGroup (HAPcocyclic), [15](#)
- Centre, [19](#)
- ChevalleyEilenbergComplex, [7](#)
- ChildClose, [23](#)
- ChildCommand, [23](#)
- ChildFunction, [24](#)
- ChildGet, [23](#)
- ChildProcess, [23](#)
- ChildPut, [23](#)
- ChildRead, [24](#)
- ChildReadEval, [24](#)
- Coclass, [11](#)
- CocycleCondition, [15](#)
- Cohomology, [8](#)
- CohomologyModule, [8](#)
- CohomologyPrimePart, [8](#)
- ComplementTopologicalSpace, [25](#)
- Compose(f,g), [27](#)
- ConcatenatedTopologicalSpace, [25](#)
- ContractTopologicalSpace, [25](#)
- CoxeterDiagramComponents, [21](#)
- CoxeterDiagramDegree, [21](#)
- CoxeterDiagramDisplay, [21](#)
- CoxeterDiagramFpArtinGroup, [21](#)
- CoxeterDiagramFpCoxeterGroup, [21](#)
- CoxeterDiagramIsSpherical, [21](#)
- CoxeterDiagramMatrix, [21](#)
- CoxeterDiagramVertices, [21](#)
- CoxeterSubDiagram, [21](#)
  
- DesuspensionFpGModule, [17](#)
- DesuspensionMtxModule, [18](#)
- Dimension, [22](#)
- DirectProductGog, [19](#)
- DirectSumOfFpGModules, [17](#)
  
- EpiCentre, [11](#)
- EquivariantChainMap, [5](#)
- EvaluateProperty, [22](#)
- EvenSubgroup, [21](#)
- ExpansionOfRationalFunction, [9](#)
  
- FpGModule, [17](#)
- FpGModuleDualBasis, [17](#)
- FpGModuleHomomorphism, [17](#)
- FpG\_to\_MtxModule, [18](#)
- Fundamental domains (HAPcryst), [14](#)
  
- GeneratorsOfFpGModule, [17](#)
- GeneratorsOfMtxModule, [18](#)
- GOuterGroup, [19](#)
- GOuterGroupHomomorphismNC, [19](#)
- GOuterHomomorphismTester, [19](#)
- GraphOfGroupsDisplay, [21](#)
- GraphOfGroupsTest, [21](#)
- GroupCohomology, [8](#)
- GroupHomology, [8](#)
- GroupOfResolution, [22](#)
  
- HAPcopyright, [27](#)
- HAPPrintTo, [23](#)
- HAPRead, [23](#)
- Homology, [8](#)

HomologyPb, 8  
 HomologyPrimePart, 8  
 HomotopyGroup, 20  
 HomotopyModule, 20  
 HomToGModule, 6  
 HomToIntegers, 6  
 HomToIntegersModP, 6  
 HomToIntegralModule, 6  
  
 IdentityAmongRelatorsDisplay, 13  
 ImageOfFpGModuleHomomorphism, 17  
 IntegralCupProduct, 10  
 IntegralRingGenerators, 10  
 IntersectionOfFpGModules, 17  
 IsAspherical, 13  
 IsAvailableChild, 23  
 IsFpGModuleHomomorphismData, 17  
 IsLieAlgebraHomomorphism, 27  
 IsSuperperfect, 27  
  
 LeibnizAlgebraHomology, 8  
 LeibnizComplex, 7  
 LeibnizQuasiCoveringHomomorphism, 12  
 Length, 22  
 LieAlgebraHomology, 8  
 LieCoveringHomomorphism, 12  
 LieEpiCentre, 12  
 LieExteriorSquare, 12  
 LieTensorCentre, 12  
 LieTensorSquare, 12  
 ListToPseudoList, 26  
 LowerCentralSeriesLieAlgebra, 6  
  
 MakeHAPManual, 27  
 Map, 22  
 MatrixToTopologicalSpace, 25  
 ModPCohomologyGenerators, 10  
 ModPCohomologyRing, 10  
 ModP RingGenerators, 10  
 ModuleAsCatOneGroup, 20  
 MooreComplex, 20  
 MultipleOfFpGModule, 17  
 MultiplyWord, 16  
  
 Negate, 16  
 NegateWord, 16  
 NextAvailableChild, 23  
 NonabelianExteriorProduct, 11  
 NonabelianSymmetricKernel, 11  
 NonabelianSymmetricSquare, 11  
 NonabelianTensorProduct, 11  
 NonabelianTensorSquare, 11  
 NormalSubgroupAsCatOneGroup, 20  
  
 OrbitPolytope, 14  
  
 ParallelList, 24  
 PathComponent, 25  
 PermToMatrixGroup, 27  
 PoincareSeries, 9  
 PoincareSeriesPrimePart, 9  
 PolytopalComplex, 14  
 PolytopalGenerators, 14  
 Prank, 9  
 PresentationOfResolution, 13  
 PrimePartDerivedFunctor, 8  
 PrintZGword, 16  
 ProjectedFpGModule, 17  
  
 RadicalOfFpGModule, 17  
 RandomHomomorphismOfFpGModules, 17  
 Rank, 17  
 RankHomologyPGroup, 8  
 RankPrimeHomology, 8  
 ReadImageAsMatrix, 25  
 ReadImageAsTopologicalSpace, 25  
 RelativeSchurMultiplier, 11  
 ResolutionAbelianGroup, 3  
 ResolutionAlmostCrystalGroup, 3  
 ResolutionAlmostCrystalQuotient, 3  
 ResolutionArtinGroup, 3  
 ResolutionAsphericalPresentation, 3  
 ResolutionBieberbachGroup (HAPcryst), 3  
 ResolutionDirectProduct, 3  
 ResolutionExtension, 3  
 ResolutionFiniteDirectProduct, 3  
 ResolutionFiniteExtension, 3  
 ResolutionFiniteGroup, 3  
 ResolutionFiniteSubgroup, 3  
 ResolutionFpGModule, 4  
 ResolutionGraphOfGroups, 3  
 ResolutionNilpotentGroup, 3  
 ResolutionNormalSeries, 3  
 ResolutionPrimePowerGroup, 3  
 ResolutionSmallFpGroup, 3  
 ResolutionSubgroup, 3

ResolutionSubnormalSeries, 3

SingularChainComplex, 25

SolutionsMatDestructive, 27

Source, 22

StandardCocycle, 15

SumOfFpGModules, 17

SumOp, 17

Syzygy, 15

Target, 22

TensorCentre, 11

TensorWithIntegers, 6

TensorWithIntegersModP, 6

TensorWithRationals, 6

TestHap, 27

ThickenedTopologicalSpace, 25

ThirdHomotopyGroupOfSuspensionB, 11

TietzeReduction, 16

TorsionGeneratorsAbelianGroup, 13

TwistedTensorProduct, 3

UpperEpicentralSeries, 11

VectorStabilizer, 14

VectorsToFpGModuleWords, 17

ViewTopologicalSpace, 25

WriteTopologicalSpaceAsImage, 25